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STANDARD DEVIATION METHOD FOR SOLVING FUZZY ASSIGNMENT PROBLEM

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ABSTRACT

Assignment Problem is universally used to solve real valued problems. In this paper, the values of the Fuzzy Assignment Problem are considered as Triangular fuzzy numbers. First, the triangular fuzzy numbers are converted into crisp values using standard deviation method. Then, the optimum schedule of the Fuzzy Assignment Problem is obtained, by usual Hungarian Method. This approach is illustrated by a numerical example.

KEYWORDS: *Fuzzy Sets, Fuzzy Triangular Numbers, Fuzzy Assignment Problem, Hungarian Method, Standard Deviation Method*

INTRODUCTION

The assignment problem is a special type of linear programming problem. Suppose, there **are** n jobs to be performed and n persons are available for doing these jobs. The assignment problem deals with the allocation of these **'**n**'** jobs to the **'**n**'** persons on one to one basis, with minimum cost or maximum profit. In earliest days, the algorithms which were used to find the optimal solution for transportation problems are applicable to solve the assignment problem. But later, a special algorithm known as Hungarian algorithm by Kuhn **[1]** was used to find the optimal solution for A.P because of its highly degenerate nature.

In assignment problem, if the parameters are fixed real numbers then it is easy to solve. But in real life situations, the time or cost for doing a job by a person or machine may vary due to different reasons so the parameters are imprecise numbers. The concept of fuzzy sets to deal with imprecision in this situation which was introduced by by Zadeh [2].

Assignment problem with fuzzy cost **(**$\overline{C}_{ij}$**)** is done by so many authors such as Chen [3] Wang [4], Lin and Wen [5]. Kathirvel.K and Balamurugan. K **[6]** proposed Robust's ranking technique for fuzzy assignment problem with fuzzy cost. But in general, there is a Hungarian method to solve the assignment problem. But if the cost values are single (crisp values), then the method is used easily. So in this paper, the fuzzy assignment problem has been converted into crisp assignment problem using standard deviation and then Hungarian algorithm has been applied to find an optimal solution.

PRELIMINARIES

Definition 2.1 (Fuzzy Set)

Let A⊆X be the given set.the membership function in the set A can be represented by

$$\mu_A(x) = \begin{cases} 1 \text{ if } x \in X \\ 0 \text{ if } x \notin X \end{cases}$$

Fuzzy set is a set having degrees of membership between 0 and 1. (i. e.,) Every function that maps X(Universe of objects) onto [0,1]. $\tilde{A}$ Is called the fuzzy set & $\mu_{\tilde{A}}$ is the degree membership of element x in fuzzy set $\tilde{A}$ such that $\mu_{\tilde{A}}(x) \in [0,1]$. The assigned value indicates the membership grade of the element in the set A. The set $\tilde{A}= \{(x, \mu_{\tilde{A}}): x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2

A Fuzzy set $\tilde{A}$, defined on universal of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

- $\tilde{A}$ is convex i.e., if for any elements x,y,z in a fuzzy set $\tilde{A}$, the relation $x<y<z$ implies that $\mu_{\tilde{A}}(y) \geq$ minimum $(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(z))$
- $\tilde{A}$ is normal, there exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}$ is piecewise continuous.

Definition 2.3

- A fuzzy number is a special case of a convex, normalized fuzzy set of the real line.
- A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0$ for all $x<0$.

Definition 2.4 A fuzzy number $\tilde{A} = (a,b,c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \leq x \leq b \\ \frac{(x-c)}{(b-c)} & b \leq x \leq c \\ 0 & x < a \text{ or } x > c \end{cases}$$

Where a, b, $c \in R$

Defuzzification

Defuzzification is the process of finding singleton value (crisp value), which represents the average value of the triangular fuzzy numbers. Here, method of standard deviation is used to defuzzify the triangular fuzzy numbers because of its simplicity and accuracy.

Assignment Problem

The assignment problem can be stated in the form of n×n cost matrix $[C_{ij}]$ of real numbers, as given in the following table:

Table 1

Jobs→ Persons↓	1	2	3	J	N
1	C_{11}	C_{12}	C_{13}	..C_{1j}..	C_{1n}
2	C_{21}	C_{22}	C_{23}	..C_{2j}..	C_{2n}
-	-	-	-	-	-
i	C_{i1}	C_{i2}	C_{i3}	..C_{ij}..	C_{in}
-	-	-	-	-	-
n	C_{n1}	C_{n2}	C_{n3}	..C_{nj}..	C_{nn}

Mathematically, assignment problem can be stated as

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\, x_{ij}$$

$$\text{Subject to } \sum_{i=1}^{n} x_{ij} = 1 \quad i = 1,2,3,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad j = 1,2,3,\ldots,n$$

$$\text{Where, } x_{ij} = \begin{cases} 1 \; if\ the\ i^{th}\ persson\ assigned\ the\ j^{th}\ job \\ 0 \; otherwise \end{cases}$$

and C_{ij} represents the cost of assignment of person i to the job j.

When the costs $\overline{C}_{ij}$ are fuzzy numbers then the total cost becomes a fuzzy number. Then, the fuzzy objective function is

$$\text{Minimize } \overline{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \overline{c}_{ij}\, x_{ij}$$

Where, $\overline{C}_{ij}$ = (a,b,c), the triangular fuzzy numbers. It cannot be minimized directly. We defuzzify the fuzzy cost coefficients into crisp ones by our proposed method.

ALGORITHMS

Hungarian Assignment Algorithm

Step: 1

Check whether the assignment problem is balanced or not. If not, add dummy row or dummy column with cost value 0 and make it as a balanced one.

Step: 2

In each row, subtract the smallest cost element from each element, so that there will be at least one zero in each row. In the same way, proceed in the column also.

Step: 3

Examine the rows successively, until a row with exactly one zero is found. Circle the zero as an assigned cell and cross out all other zero in its coulum. Proceed in this manner until all the rows have been examined. If there is more than one zero in any row, do not consider that row and pass on to the next row.

Step: 4

Repeat the procedure for the columns of the cost matrix. If there is no single zero in any row or column, then arbitrarily choose a row or column having the minimum number of zeros. Repeat the step 3 and 4 until all the zeros are either assigned or crossed out.

Step: 5

If the number of assigned cells equals the number of rows, then there is an optimal assignment schedule. If not, go to next step.

Step: 6

Draw the minimum number of horizontal or vertical lines through all the zeros as follows:

- Mark (√) to those rows where no assignment has been made.
- Mark (√) to those columns which have zeros in the marked rows.
- Mark (√) rows (not already marked) which have assignment in marked columns.
- Draw straight lines through unmarked rows and marked columns.

Step: 7

If the number of lines is equal to the number of rows or columns, the optimum solution is attained by arbitrary allocation in the position of the zeros not crossed in step: 3. if not, go to the next Step.

Step: 8

Choose the smallest element from the uncrossed elements and subtract this element from them and add the same at the point of intersection of two lines. Other elements, crossed by the lines remain unchanged.

Step: 9

Go to Step 4, and repeat the procedure till an optimum solution is attained.

Algorithm to Solve Fuzzy Assignment Problem

Step: 1

First, convert the cost values of the fuzzy Assignment Problem which are all in triangular fuzzy numbers, into crisp value, by using standard deviation.

(i. e.) If cij = (a,b,c) then the crisp value of cij which is denoted by cij' is defined by

$$cij' = \sqrt{\left[\left(\frac{a^2+b^2+c^2}{3}\right) - \left(\frac{a+b+c}{3}\right)^2\right]}$$

Step: 2

Check whether the assignment problem is balanced or not. If not, add dummy row or dummy column with cost value 0 and make it as a balanced one.

Step: 3

Using Hungarian method, find the optimum assignment schedule.

Numerical Example

To illustrate a fuzzy assignment problem, by using the proposed method:

Let us consider a fuzzy assignment problem with rows representing four workers A, B, C, D and columns representing the jobs 1,2,3,4. The cost matrix ($\bar{C}_{ij}$) is given, whose elements are triangular fuzzy numbers. The problem is to find the optimal assignment, so that the total cost of job assignment becomes Minimum.

$$\begin{bmatrix} (10,20,30) & (10,20,40) & (10,30,40) & (10,20,30) \\ (10,20,40) & (10,30,40) & (10,20,30) & (20,30,40) \\ (10,20,40) & (20,30,40) & (20,30,40) & (10,20,30) \\ (20,30,50) & (10,20,30) & (20,40,60) & (20,30,50) \end{bmatrix}$$

Solution:

The triangular fuzzy number (a, b, c) is changed to a crisp one by applying

$$\text{cij}' = \sqrt{\left[\left(\frac{a^2+b^2+c^2}{3}\right) - \left(\frac{a+b+c}{3}\right)^2\right]}.$$

C_{11}' =(10,20,30)' =8.17, C_{12}' =(10,20,40)' =12.47, C_{13}' =(10,30,40)' =12.47,

C_{14}' =(10,20,30)' =8.17, C_{21}' =(10,20,40)' =12.47, C_{22}' =(10,30,40)' =12.47,

C_{23}' =(10,20,30)' =8.17, C_{24}' =(20,30,40)' =8.17, C_{31}' =(10,20,40)' =8.17,

C_{32}' =(20,30,40)' =8.17, C_{33}' =(20,30,40)' =8.17, C_{34}' =(10,20,30)' =8.17,

C_{41}' =(20,30,50)' =12.47, C_{42}' =(10,20,30)' =8.17, C_{43}' =(20,40,60)' =16.33,

C_{43}' =(20,30,50)' =12.47

The New Cost Table is

Table 2

	I	II	III	IV
A	8.17	12.47	12.47	8.17
B	12.47	12.47	8.17	8.17
C	12.47	8.17	8.17	8.17
D	12.47	8.17	16.33	12.47

Using Hungarian Algorithm, the Optimal Allocations are

Table 3

	I	II	III	IV
A	0	8.6	8.6	0
B	0	4.3	0	0
C	0	0	0	0
D	0	0	8.16	4.3

The optimum schedule is A → I, B→III, C→IV, D→II

The optimum assignment cost is 8.17 + 8.17+ 8.17+8.17 = 32.68

Comparing the assignment cost that has been found in the above example with assignment cost calculated by existing method, it is minimum.

CONCLUSIONS

In this paper, the fuzzy costs of triangular fuzzy assignment problem has been defuzzified into crisp value by using standard deviation method, and then solved by Hungarian method. We hope that, this approach will be effective in fuzzy assignment problem involving imprecise data. Not only the triangular fuzzy numbers, we can use this method for any type of fuzzy numbers like trapezoidal, pentagon, hexagon etc.

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